MODULE 2: A.C. FUNDAMENTALS

INTRODUCTION

90% of the electrical energy used nowadays is a.c. in nature. Electrical supply used for commercial purpose is alternating. The D.C. supply has constant magnitude with respect to time. Fig. 3.1 shows the graph of such current with respect to time

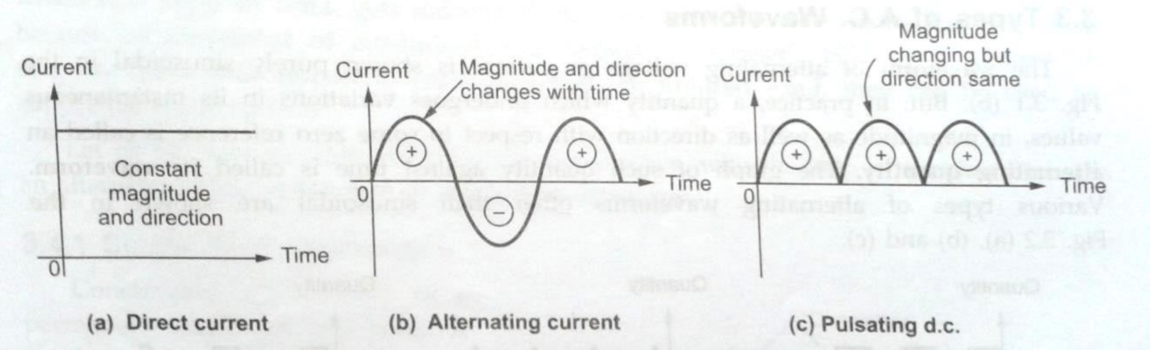


Fig 3.1

Such change in magnitude and direction is measured in terms of cycle. Each cycle of a.c. consists of two half cycles' namely positive cycle and negative cycle. Current increases in magnitude, in one particular direction, attains maximum and start decreasing, passing through zero it increases in opposite direction and behaves similarly.

In practice some waveform are available in which magnitude changes but its direction remains same as positive or negative. Such waveform is called pulsating d.c. The waveform obtained as output of full wave rectifier is an example of pulsating d.c.

ADVANTAGES OF A.C.

1. The voltages in a.c. system can be raised or lowered with the help of a device called transformer. In d.c. system, raising and lowering of voltages is not so easy.
2. As the voltages can be raised, electrical transmission at high voltages is possible. Now, higher the voltage, lesser is the current flowing through transmission line. Less the current, lesser are the copper losses and lesser is the conducting material required. This makes a.c. transmission always economical and efficient.
3. It is possible to build up high a.c. voltage; high speed a.c. generators of large capacity. The construction and cost of such generators are very low.
4. A.C. electrical motors are simple in construction, are cheaper and require less attention from maintenance point of view.
5. Whenever it is necessary, a.c. supply can be easily converted to obtain D.C. supply.

ADVANTAGES OF PURELY SINUSOIDAL WAVEFORM

6. Mathematically, it is very easy to write the equation for purely sinusoidal waveform.
7. Any other type of waveform can be resolved into a series of sine or cosine waves of fundamental and higher frequencies, sum of all these waves gives the original waveform. Hence it is always better to have sinusoidal waveform as the standard waveform.
8. The sine and cosine waves are the only waves which can pass through linear circuits containing resistance, inductance and capacitance without distortion. In case of other waveforms, there is a possibility of distortion when it passes through linear circuits.
9. The integration and derivatives of a sinusoidal function is again a sinusoidal function. This makes the analysis of linear electrical network with sinusoidal inputs, very easy.

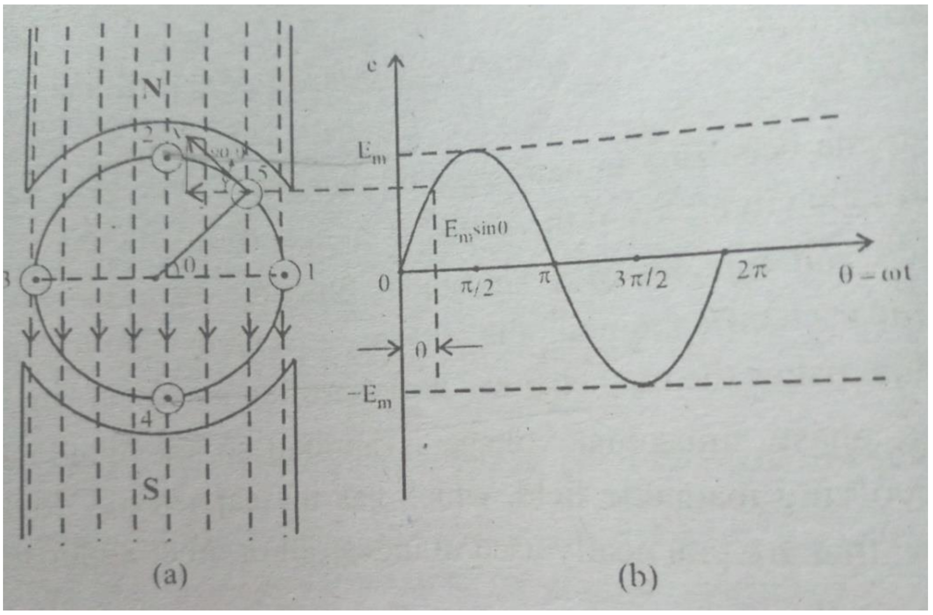
GENERATION OF SINUSOIDAL A.C. VOLTAGE

The machines which are used to generate electrical voltages are called generators. The generators which generate purely sinusoidal a.c. voltages are called alternators.

The basic principle of an alternator is the principle of electromagnetic induction. The sine wave is generated according to Faraday’s law of electromagnetic induction. It says that whenever there is a relative motion between the conductor and the magnetic field in which it is kept, an e.m.f. gets induced in the conductor. The relative motion may exist because of movement of conductors with respect to magnetic field or movement of magnetic field with respect to conductor. Such an induced e.m.f. then can be used to supply the electrical load.

Let us see how an alternator produces a sine wave, with the help of simplest form of an alternator called single turn or single loop alternator.

Construction: Consider a conductor of length ‘l’ which is placed perpendicular to the lines of magnetic flux density ‘B’ wb/m^2 produced by two poles N and S as shown in figure below.



Let the conductor rotate in a circular path with an angular velocity ω or linear velocity v from position1. When it starts rotating from position1 i.e. when the angular displacement $\Theta=0$, it moves parallel to the lines of flux and hence cuts no flux, so emf induced is zero.

At position 2, $\Theta=90^o$ conductor moves perpendicular to the lines of flux, cuts maximum flux and emf induced is E_m. Again conductor rotates through another 90^o and occupies position 3, $\Theta= 180^o$ or Π radians, it moves parallel to the lines of flux hence cuts no flux and emf induced is zero.

Now the conductor will be moved by another 90^o and occupies position 4, $\Theta= 270^o$ or $3\Pi/2$ radians, it moves perpendicular to the lines of flux, cuts max flux and hence emf induced is max.

Again the conductor will be moved by another 90^o and occupies position 1, $\Theta= 360^o$ or 2Π radians, emf induced is zero.

When the conductor is rotating from position 1 to 2 and 2 to 3 i.e. from $\Theta=0$ to $\Theta=\Pi$ (0 to 180^o) it is rotating under the influence of north pole and the direction of induced emf is taken as positive. Similarly when the conductor rotates from position 3 to 4 and 4 to 1 i.e. from $\Theta= \Pi$ to $\Theta= 2\Pi$ (180^o to 360^o) it is under the influence of south pole and hence the direction of the induced emf is taken as negative.

Consider the conductor has moved through an angle Θ and occupied position 5 as shown in figure.

The component of velocity v perpendicular to lines of flux is v $\cos(90-\Theta)$= v $\sin\Theta$

Therefore emf induced at position 5 is given by,

$$e = B\,l\,v\,\sin\Theta = E_m \sin\Theta$$

$$e = E_m \sin\omega t = E_m \sin(2\Pi f)t$$

where B l v = E_m = maximum value of the emf induced. Similar equation can be written for current also , i= $I_m \sin(2\Pi f)t$ or $I_m \sin\Theta$.

STANDARD TERMINOLOGY RELATED TO ALTERNATING QUANTITY

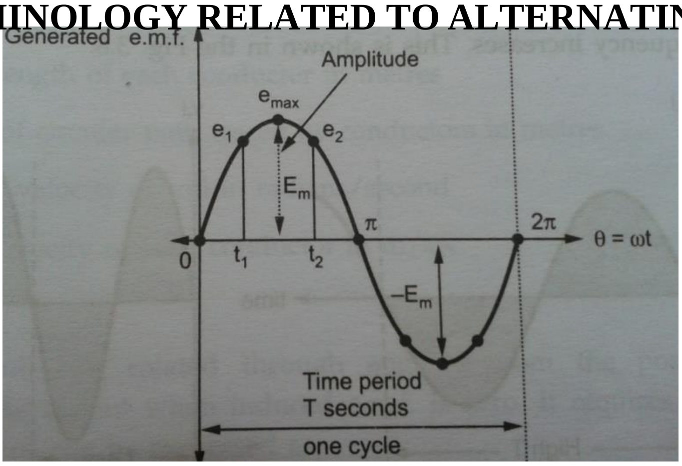


Fig Waveform of an alternating e.m.f

Instantaneous value

The value of an alternating quantity at a particular instant is known as **instantaneous value**.

E.g. e_1 and e_2 are the instantaneous value of an alternating emf at the instant t_1 and t_2 respectively.

Waveform

The graph of instantaneous values of an alternating quantity plotted against time is called its **waveform.**

Cycle

Each repetition of a set of positive and negative instantaneous values of an alternating quantity is called a **cycle**.

Time Period (T)

Time taken by an alternating quantity to complete its one cycle is known as its **time period** denoted by T seconds. After every T seconds, the cycle of an alternating quantity repeats.

Frequency (f)

The number of cycles completed by an alternating quantity per second is known as its frequency. It is denoted by f and is measured in cycles/second which is known as hertz, denoted as Hz.

$$\mathbf{f = \frac{1}{T}\ Hz}$$

3.5.1 Amplitude

The maximum value attained by an alternating quantity during positive or negative half cycle is called its amplitude. It is denoted as E_m or I_m.

3.5.2 Angular Frequency (ω)

It is the frequency expressed in electrical radians per second. As one cycle of an alternating quantity corresponds to 2π radians, the angular frequency can be expressed as (2π * cycles/sec). its unit is radians/sec.

$$\mathbf{\omega = 2\,\pi\,f\ radians/sec}$$

EFFECTIVE VALUE OR R.M.S VALUE

The ***effective value or r.m.s. value*** *of an alternating current is given by that steady current which, when flowing through a given circuit for a given time produces the same amount of heat as produced by the alternating current, which when flowing through the same circuit for the same time.*

Consider sinusoidally varying alternating current and square of this current as shown in figure 3.3

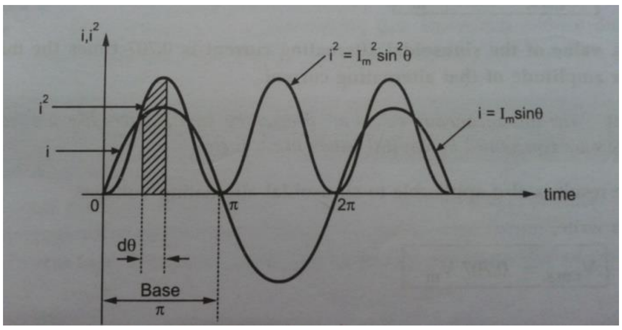


Fig 3.3 Waveform of current and square of the current

The current $i = I_m \sin\theta$ while

Square of the current $i^2 = I_m^2 \sin^2\theta$

Area of curve over half a cycle can be calculated by considering an interval dθ as shown.

Area of square curve over half cycle= $\int_0^{\pi} i^2\, d\theta$ and length of the base is π

Therefore Average value of square of the current over half cycle

$$= \frac{\text{area of curve over half cycle}}{\text{length of base over half cycle}} = \frac{\int_0^{\pi} i^2\, d\theta}{\pi}$$

$$= \frac{1}{\pi}\int_0^{\pi} i^2\, d\theta = \frac{1}{\pi}\int_0^{\pi} I_m^2 \sin^2\theta\, d\theta = \frac{I_m^2}{\pi}\int_0^{\pi} \frac{1-\cos 2\theta}{2} d\theta$$

$$= \frac{I_m^2}{\pi}\left[\theta - \frac{\sin 2\theta}{2}\right]_o^{\pi} = \frac{I_m^2}{2\pi}[\pi]$$

$$= \frac{I_m^2}{2}$$

Hence, root mean square value i.e. r.m.s. value can be calculated as

$$I_{r.m.s} = \sqrt{\text{mean or average value of current}} = \sqrt{\frac{I_m^2}{2}}$$

$$= \frac{I_m}{\sqrt{2}}$$

$$\mathbf{I_{r.m.s} = 0.707 I_m}$$

AVERAGE VALUE

The **average value** of an alternating quantity is defined as that value which is obtained by averaging all the instantaneous values over a period of half cycle.

For a symmetrical a.c., the average value over a complete cycle is zero as both positive and negative half cycles are identical. Hence, the average value is defined for half cycle only.

Average value can also be expressed by that steady current which transfers across any circuit, that same amount of charge as is transferred by that alternating current during the same time.

For an unsymmetrical a.c., the average value must be obtained for one complete cycle but for symmetrical a.c. like sinusoidal; it is to be obtained for half cycle.

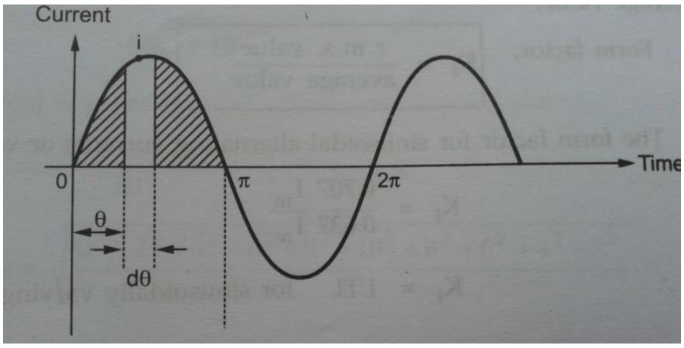


Fig 3.4 Average value of an alternating current

Consider sinusoidally varying current, $I = I_m \sin\theta$

Consider the elementary interval of instant $d\theta$ as shown in figure. The average instantaneous value of current in this interval say I.

The average value can be obtained by taking ratio of area under curve over half cycle to length of the base for half cycle.

$$I_{av} = \frac{\text{area under curve for half cycle}}{\text{length of base over half cycle}}$$

$$= \frac{\int_0^{\pi} i \, d\theta}{\pi}$$

$$= \frac{1}{\pi}\int_0^{\pi} i \, d\theta = \frac{1}{\pi}\int_0^{\pi} I_m \sin\theta \, d\theta = \frac{I_m}{\pi}\int_0^{\pi} \sin\theta \, d\theta$$

$$= \frac{I_m}{\pi}[-\cos\theta]_0^{\pi} = \frac{I_m}{\pi}[-\cos\pi + \cos\theta]$$

$$= \frac{I_m}{\pi}[2]$$

$$= \frac{2I_m}{\pi}$$

$$\mathbf{I_{av} = 0.637 I_m}$$

FORM FACTOR (K_f)

The form factor of an alternating quantity is defined as the ratio of r.m.s. value to the average value.

$$K_f = \frac{\text{r.m.s value}}{\text{average value}}$$

The form factor for sinusoidal alternating currents or voltages can be obtained as,

$$K_f = \frac{0.707\, I_m}{0.637\, I_m}$$

$\mathbf{K_f = 1.11}$ for sinusoidally varying quantity

CREST OR PEAK FACTOR(K_P)

The peak value of an alternating quantity is defined as the ratio of maximum value to the r.m.s. value.

$$K_P = \frac{\text{maximum value}}{\text{r.m.s. value}}$$

The peak factor for sinusoidal alternating currents or voltages can be obtained as

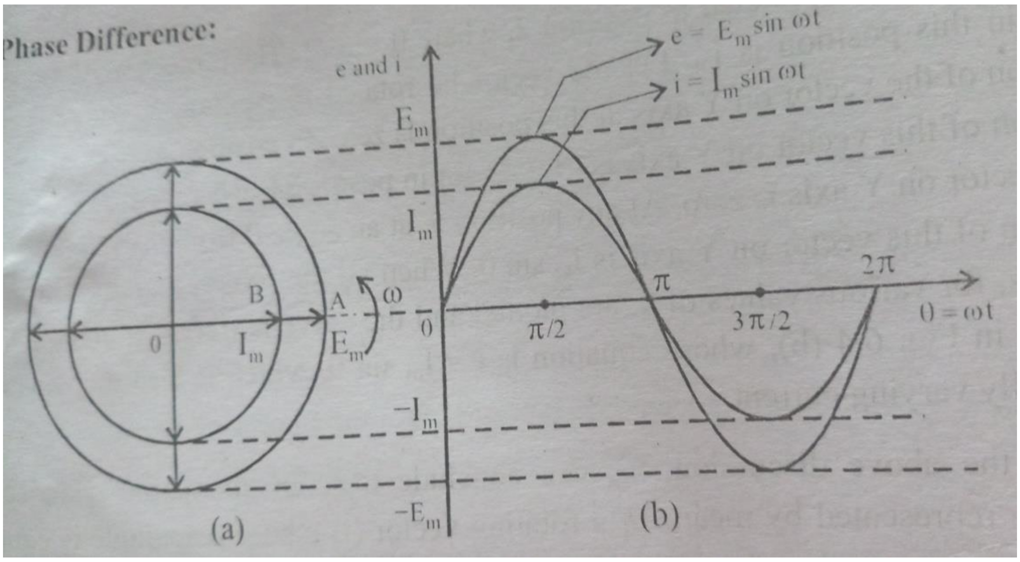
$$K_P = \frac{I_m}{0.707\, I_m} = 1.414$$

PHASE OF AN ALTERNATING QUANTITY

Phase of an alternating quantity is the angle through which the rotating vector representing the alternating quantity has rotated through from the reference axis.

Phase difference between the two alternating quantities is the angle difference between the two rotating vectors representing the two alternating quantities.

Case 1:



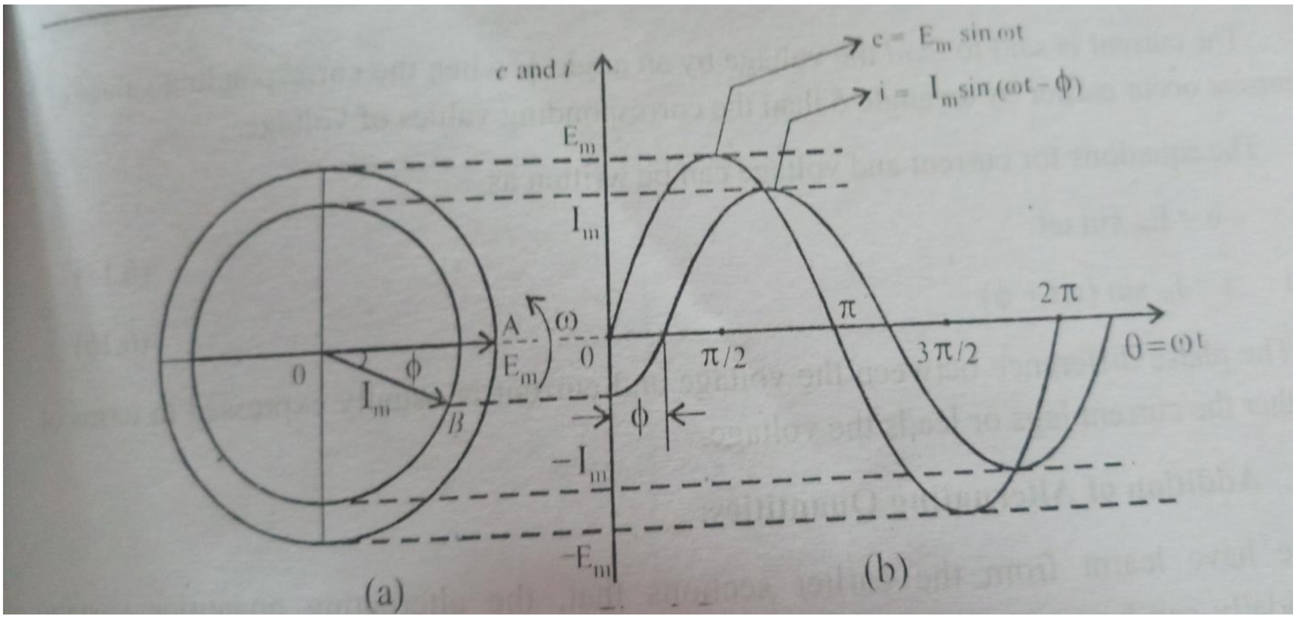
The rotating vector OA represents the alternating voltage and OB represents the alternating current. Both of them rotate together with an angular velocity ω and phase difference is zero.

Two quantities are said to be in phase with each other when their corresponding values occur at the same time.

The equations for voltage and current are $e = E_m \sin\omega t$

$$i = I_m \sin\omega t$$

Case 2: Lagging



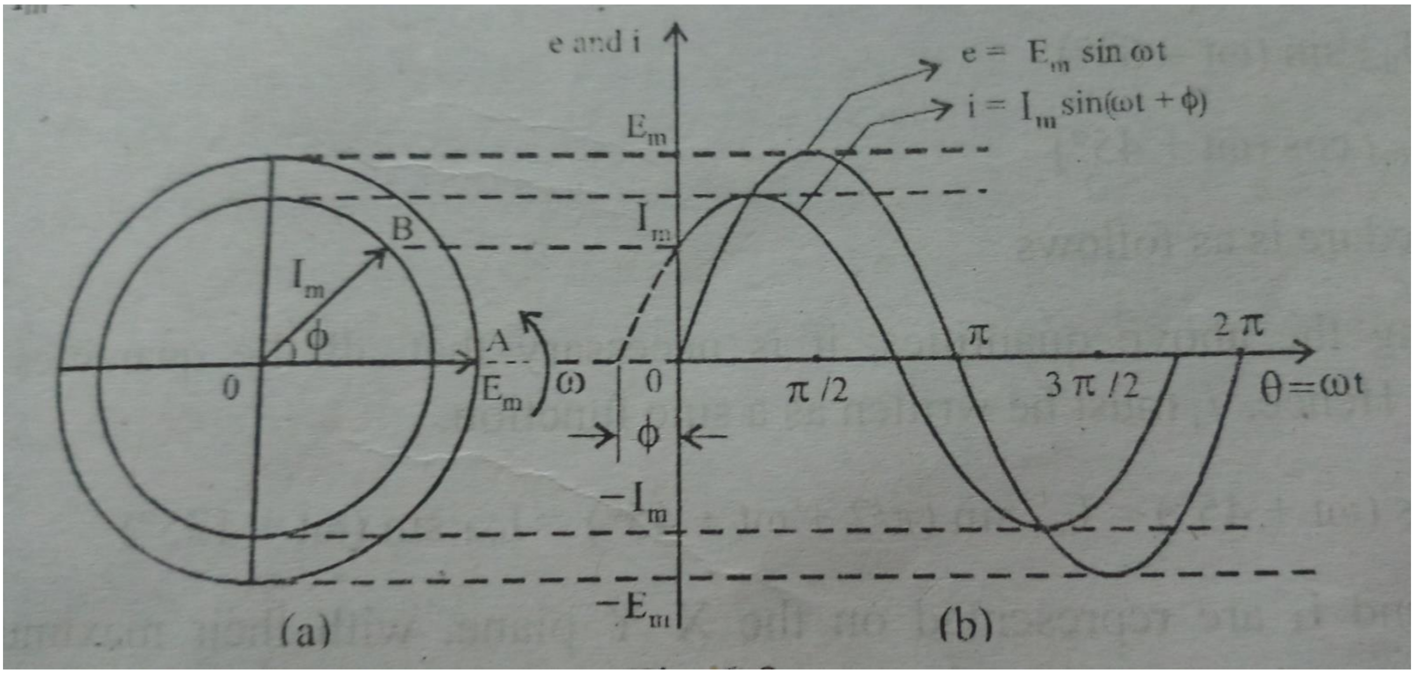
Vectors OA and OB rotate with an angle difference of ϕ, the current vector always lags the voltage vector by an angle ϕ.

Equations for voltage and current are

$e = E_m \sin\omega t$

$i = I_m \sin(\omega t - \phi)$

Case 3: Leading



Vectors OA and OB rotate with an angle difference of ϕ, current vector leads the voltage vector by an angle ϕ.

Equations for current and voltage are, $e= E_m \sin\omega t$

$$i= I_m \sin(\omega t+\phi)$$

SINGLE PHASE A.C. CIRCUITS

The resistance, inductance and capacitance are three basic elements of any electrical network. In order to analyze any electric circuit, it is necessary to understand the following three cases,

1. A.C. through pure resistive circuit.
2. A.C. through pure inductive circuit.
3. A.C. through pure capacitive circuit.

In each case, it is assumed that a purely sinusoidal alternating voltage given by the equation $v = V_m \sin \omega t$ is applied to the circuit. The equation of the current, power and phase shift is developed in each case. The voltage applied having zero phase angle is assumed reference while plotting the phasor diagram in each case.

A.C. THROUGH PURE RESISTANCE

Consider a simple circuit consisting of a pure resistance 'R' ohm connected across a voltage $v = V_m \sin \omega t$.

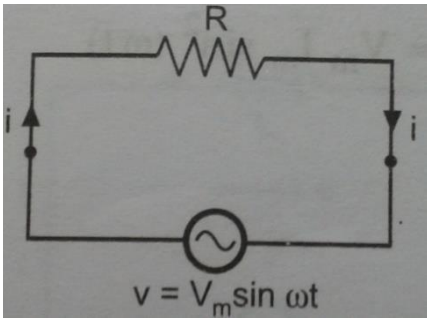


Fig 3.5 Pure resistive circuit

According to ohms law, we can find the equation for the current i as

$$i = \frac{v}{R} = \frac{V_m \sin\omega t}{R} = \left(\frac{V_m}{R}\right) \sin\omega t$$

This is the equation giving instantaneous value of the current.

Comparing this with standard equation,

$$i = I_m \sin(\omega t + \emptyset)$$

$$I_m = \frac{V_m}{R} \text{ and } \emptyset = 0$$

So, maximum value of alternating current, i is $I_m = \frac{V_m}{R}$ while, as Ø=0, it indicates that it is in phase with the voltage applied. There is no phase difference between the two. The current is going to achieve its maximum and zero whenever voltage is going to achieve its maximum and zero values.

The waveform of voltage and current and the corresponding phasor diagram is shown in fig 3.6

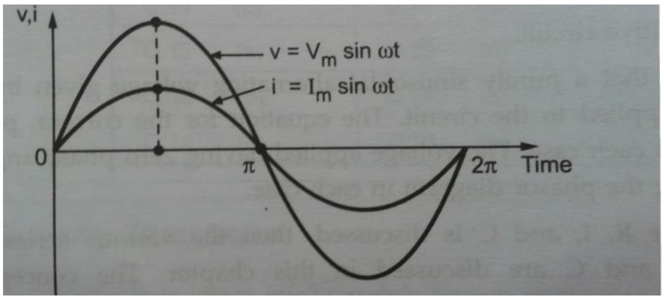


Fig 3.6 Phasor diagram of resistive circuit

Power

The instantaneous power in a.c. circuits can be obtained by taking product of the instantaneous values of current and voltage.

$$P = v*i = V_m \sin \omega t * I_m \sin \omega t = V_m\ I_m\ \sin^2\omega t$$
$$= \frac{V_m\ I_m}{2}(1 - \cos 2\omega t)$$
$$= \frac{V_m\ I_m}{2} - \frac{V_m\ I_m}{2} \cos 2\omega t$$

From the above equation, it is clear that the instantaneous power consist of two components,

1. Constant power component $(\frac{V_m\ I_m}{2})$
2. Fluctuating component $[\frac{V_m\ I_m}{2} \cos 2\omega t]$ having frequency, double the frequency of applied voltage.

Now, the average value of the fluctuating component of double frequency is zero, over one complete cycle. So, average power consumption over one cycle is equal to the constant power component i.e. $(\frac{V_m\ I_m}{2})$

$$P_{av} = \frac{V_m\ I_m}{2} = \frac{V_m}{\;} * \frac{I_m}{\;}$$

P_{av} = V_{rms}* I_{rms} watts

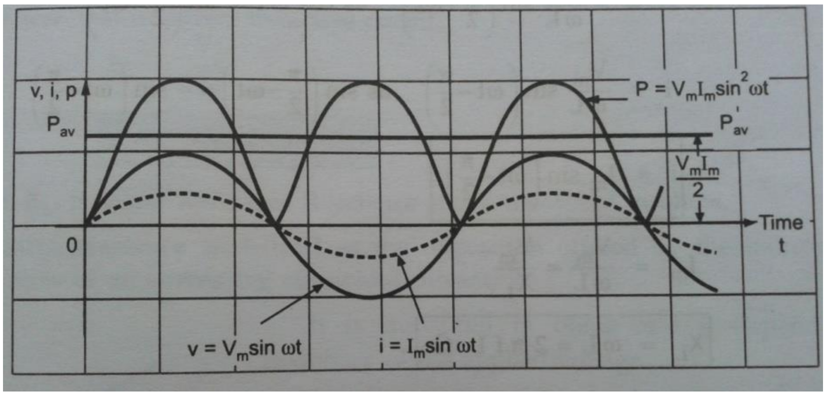


Fig 3.7 v,i, p for purely resistive circuit

A.C. THROUGH PURE INDUCTANCE

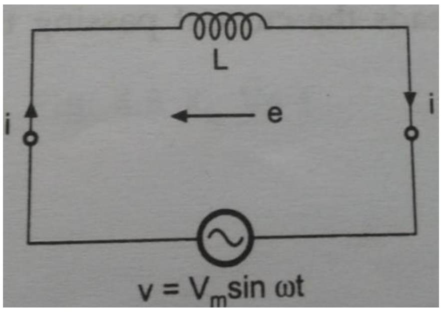


Fig 3.8 purely inductive circuit

Consider a simple circuit consisting of a pure inductance of L henries, connected across a voltage given by the equation, $v = V_m \sin \omega t$.

Pure inductance has zero ohmic resistance. Its internal resistance is zero. The coil has pure inductance of L henries.

When alternating quantity i flows through inductance 'L', it sets up an alternating magnetic field around the inductance. This changing flux links the coil and due to self inductance, emf gets induced in the coil. This emf opposes the applied voltage.

The self induced emf in the coil is given by, $e = -L \frac{di}{dt}$

At all instant, the applied voltage v is equal and opposite to the self induced emf

$$v = -e = -\left(-L \frac{di}{dt}\right)$$

$$v = L \frac{di}{dt}$$

$$V_m \sin \omega t = L \frac{di}{dt}$$

$$di = \frac{V_m}{L} \sin \omega t$$

$$i = \int di = \int \frac{V_m}{L} \sin \omega t = \frac{V_m}{L} \left(\frac{-\cos \omega t}{\omega}\right)$$

$$= -\frac{V_m}{\omega L} \sin\left(\frac{\pi}{2} - \omega t\right) \qquad \text{as } \cos \omega t = \sin\left(\frac{\pi}{2} - \omega t\right)$$

$$i = \frac{V_m}{\omega L} \sin\left(\omega t - \frac{\pi}{2}\right)$$

$$\mathbf{i = I_m \sin\left(\omega t - \frac{\pi}{2}\right)}$$

where $I_m = \frac{V_m}{\omega L} = \frac{V_m}{X_L}$

where $X_L = \omega L = 2 \pi f L \ \Omega$

The term, X_L is called **Inductive Reactance** and is measured in **ohms**

The above equation clearly shows that the current is purely sinusoidal and having phase angle of $-\frac{\pi}{2}$ radians i.e. -90°. This means that **the current lags voltage applied by 90°.**

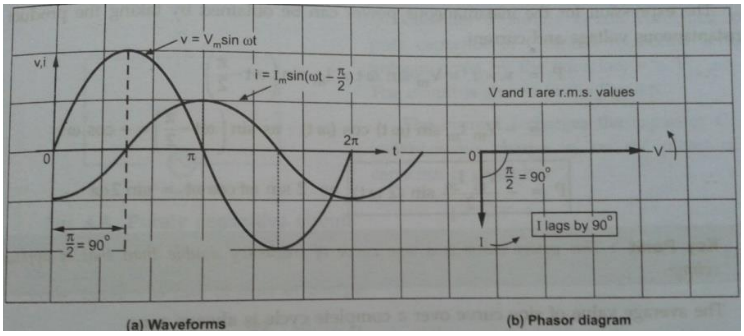


Fig 3.9

Power

The instantaneous power in a.c. circuits can be obtained by taking product of the instantaneous values of current and voltage.

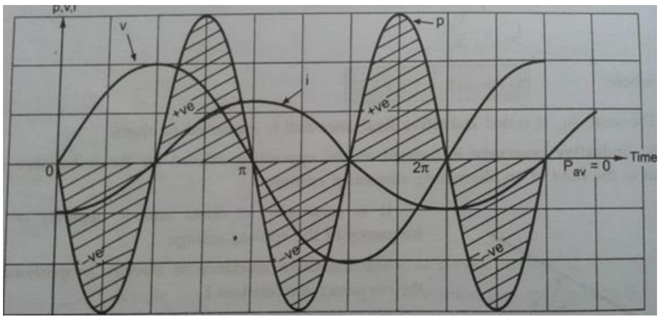
$$P = v*i = V_m \sin \omega t * I_m \sin(\omega t - \frac{\pi}{2})$$

$$= - V_m I_m \sin(\omega t) \cos(\omega t)$$

$$\mathbf{P} = -\frac{\mathbf{V_m I_m}}{\mathbf{2}} \mathbf{\sin(2\omega t)}$$

The average value of sine curve over a complete cycle is always zero

$$P = \int_0^{2\pi} \frac{V_m I_m}{2} \sin(2\omega t)\, d(\omega t) = 0$$



It can observed from it that when power curve is positive, energy gets stored in the magneticfield established due to increasing current while during negative power curve, this power is returned back to the supply. The areas of positive loop and negative loop are exactly same and hence, average powerconsumption is zero.

A.C. THROUGH PURE CAPACITANCE

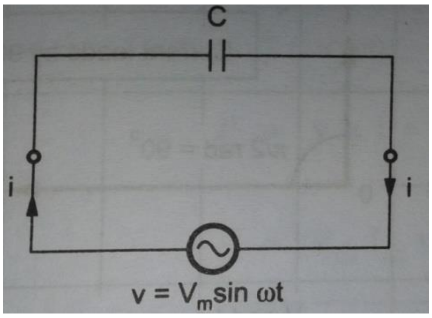


Fig 3.11

Consider a simple circuit consisting of a pure capacitor of C-farads, connected across a voltage given by the equation $v = V_m \sin \omega t$.

The current i charges the capacitor C. The instantaneous charge 'q' on the plates of the capacitor is given by

$$q = C\,v$$

Therefore $\quad q = C\,V_m \sin \omega t$

Now, current is rate of flow of charge

$$i = \frac{dq}{dt} = \frac{d}{dt}(CV_m \sin \omega t)$$

$$i = CV_m \frac{d}{dt}(\sin \omega t) = CV_m\, \omega \cos(\omega t)$$

$$i = \frac{V_m}{\frac{1}{\omega C}} \sin(\omega t + \frac{\pi}{2})$$

$$\mathbf{I_m \sin(\omega t + \frac{\pi}{2})}$$

where $I_m = \frac{V_m}{X_c}$ where $X_c = \frac{1}{\omega C} = \frac{1}{2\pi f C}\ \Omega$

The term X_c is called **capacitive reactance** and measured in **ohms.**

The above current equation clearly shows that the current is purely sinusoidal and having phase angle of $+\frac{\pi}{2}$ radians i.e. +90°.

This means **current leads voltage applied by 90°.** The positive sign indicates leading nature of the current.

Fig 3.12 shows waveform of voltage and current and the corresponding phasor diagram. The current waveform starts earlier by 90° in comparison with voltage waveform. When voltage is zero, the current has positive maximum value.

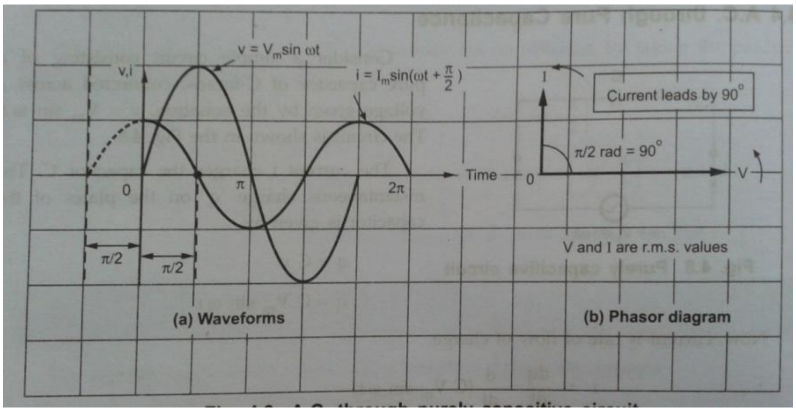


Fig 3.12

POWER

The instantaneous power in a.c. circuits can be obtained by taking product of the instantaneous values of current and voltage.

$$P = v*i = V_m \sin \omega t * I_m \sin(\omega t + \frac{\pi}{2})$$

$$= V_m I_m \sin(\omega t) \cos(\omega t)$$

$$\mathbf{P} = \frac{V_m I_m}{2} \sin(2\omega t)$$

The average value of sine curve over a complete cycle is always zero

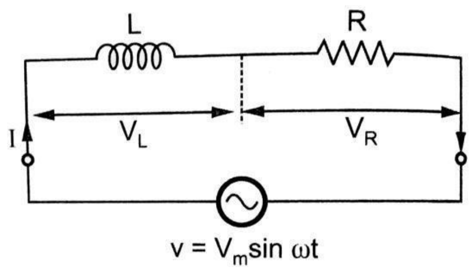
$$P = \int_0^{2\pi} \frac{V_m I_m}{2} \sin(2\omega t) \; d(\omega t) = 0$$

Fig 3.14 shows waveforms of current, voltage and power. It can observed from the figure that when power curve is positive, in practice, an electrostatic energy gets storedin the capacitor during its charging while the negative power curve represents that the energy stored is returned back to the supply during its discharging. The areas of positive and negative loops are exactly the same and hence, average power consumption is zero.

SINGLE PHASE AC CIRCUITS

A.C. THROUGH SERIES R-L CIRCUIT

Consider a circuit consisting of pure resistance R ohms connected in series with a pure inductance of L Henries. The series combination is connected across a.c. supply given by $v = V_m \sin\omega t$.



Circuit draws a current I then there are two voltage drops,

a) Drop across pure resistance, $V_R = I R$

b) Drop across pure inductance, $V_L = I X_L$ where $X_L = 2 \pi f L$

I= r.m.s. value of current drawn

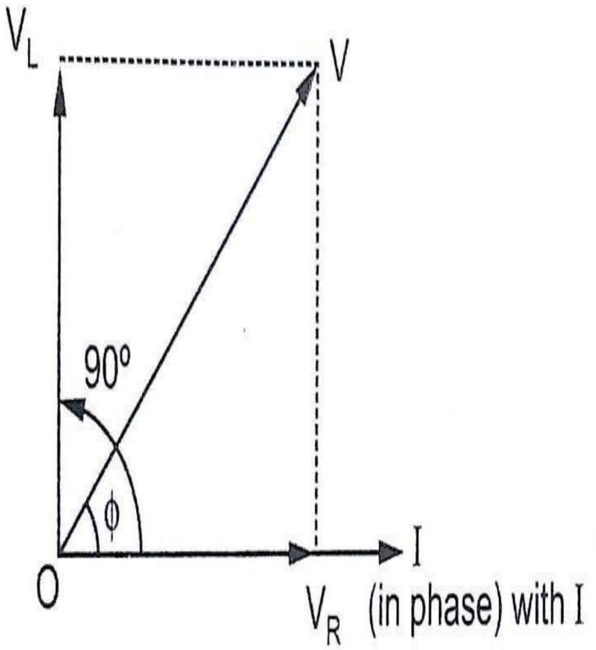
V_R, V_L= r.m.s. values of voltage drops

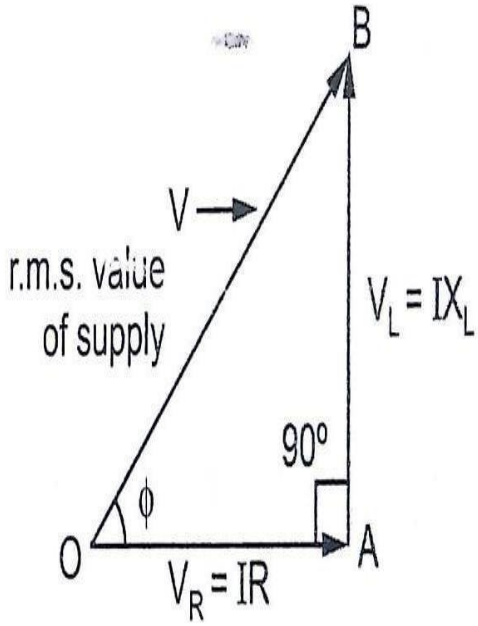
The Kirchhoff's law can be applied to the a.c. circuit but only the point to remember is the addition of voltages should be vector addition.

Therefore $\bar{V} = \bar{V}_R + \bar{V}_L$

Therefore $\bar{V} = \bar{I}R + \bar{I}\bar{X}_L$

Let us draw the phasor diagram for the above case





From the voltage triangle

$V = \sqrt{(V_R)^2 + (V_L)^2} = \sqrt{(IR)^2 + (IX_L)^2} = I\sqrt{(R)^2 + (X_L)^2}$

$V = IZ$

Where $Z = \sqrt{(R)^2 + (X_L)^2}$

The impedance Z is measured in ohms.

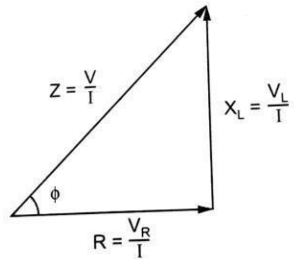
IMPEDANCE

Impedance is defined as the opposition of circuit to the flow of alternating current. It is denoted by Z and its unit is ohms.

For the R-L series circuit, it can observed from the phasor diagram that the current lags behind the applied voltage by an angle Ø. From the voltage triangle, we can write

$$\text{Tan } \varnothing = \frac{V_L}{V_R} = \frac{X_L}{R}, \qquad \cos\varnothing = \frac{V_R}{V} = \frac{R}{Z}, \qquad \text{Sin } \varnothing = \frac{V_L}{V} = \frac{X_L}{Z}$$

If all the sides of the voltage triangle are divided by current, we get a triangle called impedance triangle.



From the impedance triangle, we can see that the X component of impedance is R and isgiven by R= Zcosφ

And Y component of impedance is X_L and is given by X_L=Zsinφ

In rectangular form the impedance is denoted as

$$Z = R + jX_L$$

While in the polar form, it is denoted as

$$Z = |Z| \angle \varphi$$

Where $|Z| = \sqrt{R^2 + X_L^2}$ and $\varphi = tan^{-1}\frac{X_L}{R}$

POWER AND POWER TRIANGLE

The expression for the current in a series R-L circuit

$$i = I_m \sin(\omega t - \phi) \text{ as current lags voltage.}$$

The power is product of instantaneous values of voltage and current,

$$\therefore \quad P = v \times i = V_m \sin \omega t \times I_m \sin(\omega t - \phi) = V_m I_m [\sin(\omega t) \cdot \sin(\omega t - \phi)]$$

$$= V_m I_m \left[\frac{\cos(\phi) - \cos(2\omega t - \phi)}{2}\right] = \frac{V_m I_m}{2} \cos \phi - \frac{V_m I_m}{2} \cos(2\omega t - \phi)$$

Now, the second term is cosine term whose average value over a cycle is zero. Hence, average power consumed is,

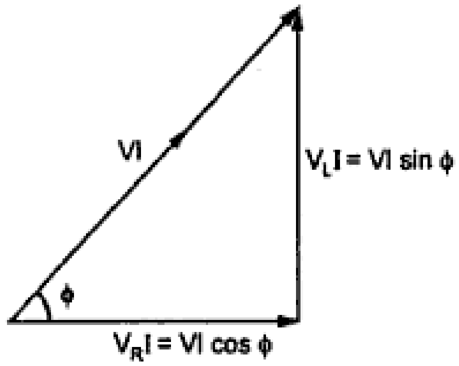
$$P_{av} = \frac{V_m I_m}{2} \cos \phi = \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}} \cos \phi$$

$$\therefore \quad \boxed{P = V I \cos \phi \ \text{ watts}} \quad \text{where V and I are r.m.s. values}$$

If we multiply voltage equation by current I, we get the power equation.

$$\overline{VI} = \overline{V_R I} + \overline{V_L I}$$

$$\therefore \quad \overline{VI} = \overline{V \cos \phi I} + \overline{V \sin \phi I}$$



From this equation, power triangle can be obtained as shown in the Fig.

So, three sides of this triangle are,

1) VI, 2) VI cos ϕ 3) VI sin ϕ

These three terms can be defined as below.

APPARENT POWER

It is defined as the product of r.m.s. value of voltage (V) and current (I). It is denoted by S.

$$\boxed{S = VI \quad VA}$$

It is measured in unit volt-amp (VA) or kilo volt-amp (kVA).

REAL OR TRUE POWER (P)

It is defined as the product of the applied voltage and the active component of the current.

$$\boxed{P = VI\cos\phi \quad \text{watts}}$$

It is real component of the apparent power. It is measured in unit watts (W) or kilowatts (kW).

Reactive Power (Q)

It is defined as product of the applied voltage arid the reactive component of the current.

It is also defined as imaginary component of the apparent power. It is represented by ‘Q’ and it is measured in unit volt-amp reactive (VAR) or kilovolt-amp reactive (kVAR).

$$\boxed{Q = VI\sin\phi \quad VAR}$$

POWER FACTOR (COSΦ)

It is defined as factor by which the apparent power must be multiplied in order to obtain the true power.

It is the ratio of true power to apparent power.

$$\boxed{\text{Power factor} = \frac{\text{True Power}}{\text{Apparent Power}} = \frac{VI\cos\phi}{VI} = \cos\phi}$$

The numerical value of cosine of the phase angle between the applied voltage and the current drawn from the supply voltage gives the power factor. It cannot be greater than 1.

It is also defined as the ratio of resistance to the Impedance $\boxed{\cos\phi = \frac{R}{Z}}$

If current lags voltage power factor is said to be lagging. If current leads voltage power factor is said to be leading.

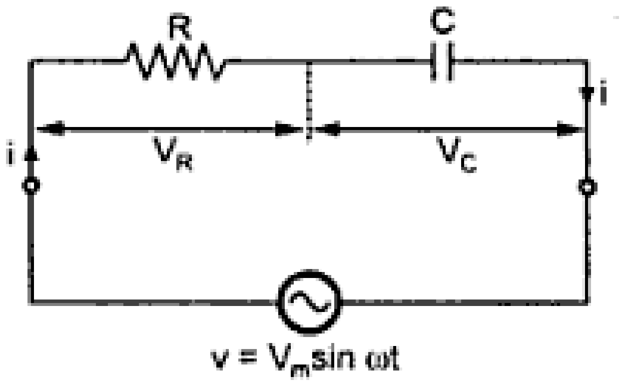
So, for pure inductance, the power factor is cos (90°) i.e. zero lagging while for pure capacitance, the power factor is cos (90°) i.e. zero but leading. For purely resistive circuit voltage and current are in phase i.e. φ= 0. Therefore, power factor is cos (0°) = 1. Such circuit is called unity power factor circuit.

$$\boxed{\text{Power factor} = \cos\phi}$$

φ is the angle between supply voltage and current.

A.C. THROUGH SERIES R-C CIRCUIT

Consider a circuit consisting of pure resistance R-ohms and connected in series with a pure capacitor of C-farads as shown in the Fig. The series combination is connected across ac. supply given by $v = V_m \sin \omega t$.



Circuit draws a current I, then there are two voltage drops,

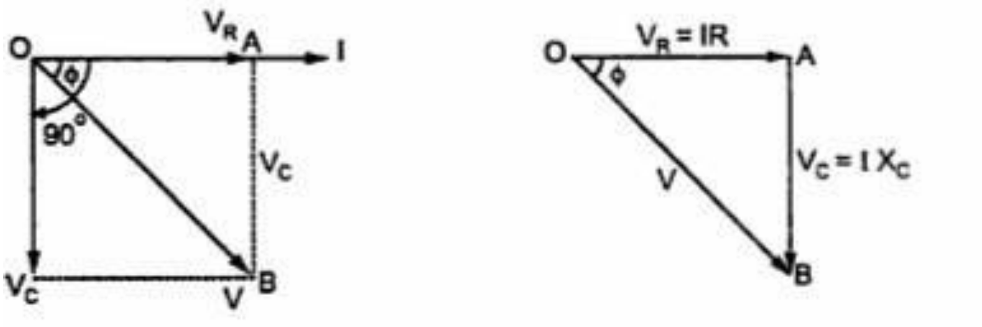
a) Drop across pure resistance, $V_R = I\,R$
b) Drop across pure inductance, $V_C = I\,X_c$ where $X_c = \frac{1}{2\pi f C}$

I= r.m.s. value of current drawn
V_R, V_c= r.m.s. values of voltage drops

The Kirchhoff's voltage law can be applied to get,

$$V = \overline{V_R} + \overline{V_C}$$
$$\overline{V} = \overline{IR} + \overline{IX_C}$$

Let us draw the phasor diagram. Current I is taken as reference as it is common to both the elements.



From the voltage triangles,

$$V = \sqrt{(V_R)^2 + (V_C)^2} = \sqrt{(IR)^2 + (I X_C)^2} = I\sqrt{(R)^2 + (X_C)^2}$$

$\therefore$ $$V = I Z$$

Where $\boxed{Z = \sqrt{(R)^2 + (X_C)^2}}$ is the impedance of the circuit.

Impedance

Similar to R-L series circuit, in this case also, the impedance is nothing but opposition to the flow of alternating current. It is measured in ohms given by $Z = \sqrt{R^2 + {X_c}^2}$ where $X_c = \frac{1}{2\pi f C}\Omega$ called capacitive reactance. |

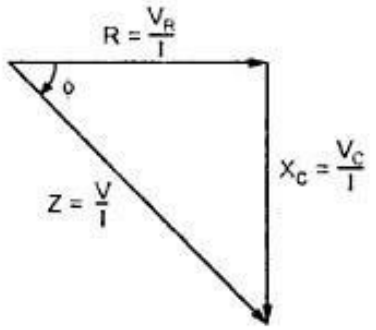
In R-C series circuit, current leads voltage by angle ϕ or supply voltage V lags current I by angle ϕ as shown in the phasor diagram in Fig. 7.20.

From voltage triangle, we can write,

$$\tan\phi = \frac{V_C}{V_R} = \frac{X_C}{R}, \quad \cos\phi = \frac{V_R}{V} = \frac{R}{Z}, \quad \sin\phi = \frac{V_C}{V} = \frac{X_C}{Z}$$

If all the sides of the voltage triangle are divided by the current, we get a triangle called **impedance triangle.**

Two sides of the triangle are 'R' and 'X_C' and the third side is impedance 'Z'.



Impedance triangle

The X component of impedance is R and is given by

$$R = Z\cos\phi$$

and Y component of impedance is X_C and is given by

$$X_C = Z\sin\phi$$

But, as direction of the X_C is the negative Y direction, the rectangular form of the impedance is denoted as,

$$\boxed{Z = R - jX_C\ \Omega}$$

While in polar form, it is denoted as,

$$\boxed{Z = |Z| \angle -\phi\ \Omega}$$

$$Z = R - jX_C = |Z| \angle -\phi$$

where $\quad |Z| = \sqrt{R^2 + X_C^2},\ \phi = \tan^{-1}\left[\frac{-X_C}{R}\right]$

Power and Power Triangle

The current leads voltage by angle ϕ, hence its expression is,

$$i = I_m \sin(\omega t + \phi) \text{ as current leads voltage}$$

The power is the product of instantaneous values of voltage and current.

$$\therefore \quad P = v \times i = V_m \sin\omega t \times I_m \sin(\omega t + \phi)$$

$$= V_m I_m [\sin(\omega t) . \sin(\omega t + \phi)]$$

$$= V_m I_m \left[\frac{\cos(-\phi) - \cos(2\omega t + \phi)}{2}\right]$$

$$= \frac{V_m I_m \cos\phi}{2} - \frac{V_m I_m}{2}\cos(2\omega t + \phi) \qquad \text{as } \cos(-\phi) = \cos\phi$$

Now, second term is cosine term whose average value over a cycle is zero. Hence, average power consumed by the circuit is,

$$P_{av} = \frac{V_m I_m}{2}\cos\phi = \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}}\cos\phi$$

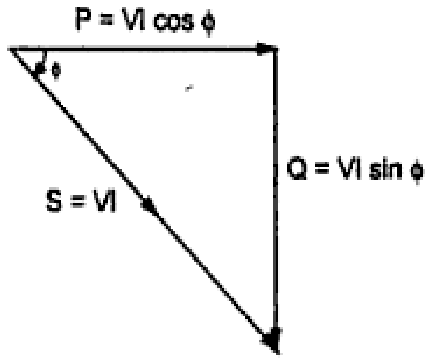
$$\therefore \quad \boxed{P = V I \cos\phi \quad \text{watts}} \quad \text{where V and I are r.m.s. values}$$

If we multiply voltage equation by current I, we get the power equation,

$$\overline{VI} = \overline{V_R I} + \overline{V_C I}$$

$$\therefore \quad \overline{VI} = \overline{VI\cos\phi} + \overline{VI\sin\phi}$$

Hence, the power triangle can be shown as in the Fig.



Thus, the various powers are,

Apparent power,	$S = V I$	VA
True or average power,	$P = V I \cos\phi$	W
Reactive power,	$Q = V I \sin\phi$	VAR

Remember that, X_L term appears positive in Z.

$$Z = R + j X_L = |Z| \angle \phi \qquad \phi \text{ is positive for inductive Z}$$

While X_C term appears negative in Z.

$$Z = R - j X_C = |Z| \angle -\phi \qquad \phi \text{ is negative for capacitive Z}$$

For any single phase a.c. circuit, the average power is given by,

$$P = V I \cos\phi \quad \text{watts}$$

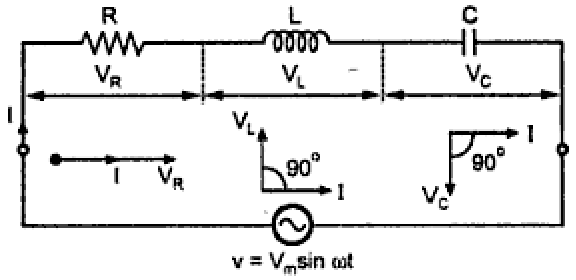
Where V, I are r.m.s. values

$\cos\phi$ = Power factor of circuit

$\cos\phi$ is lagging for inductive circuit and $\cos\phi$ is leading for capacitive circuit.

A.C. THROUGH SERIES R-L-C CIRCUIT

Consider a circuit consisting of resistance R ohms pure Inductance L henries and capacitance C farads connected in series with each other across a.c. supply. The circuit s shown below.



The a.c. supply is given by, $v = V_m \sin \omega t$

The circuit draws a current I. Due to current I, there are different voltage drops across R, L and C which is given by.

a) Drop across resistance R is $V_R = I R$

b) Drop across inductance L is $V_L = I X_L$

c) Drop across capacitance C is $V_C = I X_C$

The values of I, V_R, V_L and V_C are r.m.s. values

The characteristics of three drops are,

a) V_R is in phase with current I.

b) V_L leads current I by 90°.

c) V_C lags current I by 90°.

According to Kirchhoff's laws, we can write,

$$\bar{V} = \bar{V}_R + \bar{V}_L + \bar{V}_C \quad \text{... Phasor addition}$$

Let us see the phasor diagram. Current I is taken as reference as it is common to all the elements.

Following are the steps to draw the phasor diagram :

1) Take current as reference.
2) V_R is in phase with I.
3) V_L leads current I by 90°.
4) V_C lags current I by 90°.
5) Obtain the resultant of V_L and V_C. Both V_L and V_C are in phase opposition (180° out of phase).
6) Add that with V_R by law of parallelogram to get the supply voltage.

The phasor diagram depends on the conditions of the magnitudes of V_L and V_C which ultimately depends on the values of X_L and X_C. Let us consider the different cases.

1 $X_L > X_C$

When $X_L > X_C$, obviously, I X_L i.e. V_L is greater than I X_C i.e. V_C. So, resultant of V_L and V_C will be directed towards V_L i.e. leading current I. Current I will lag the resultant of V_L and V_C i.e. $(V_L - V_C)$.

The circuit is said to be **inductive** in nature. The phasor sum of V_R and $(V_L - V_C)$ gives the resultant supply voltage, V. This is shown in the Fig.

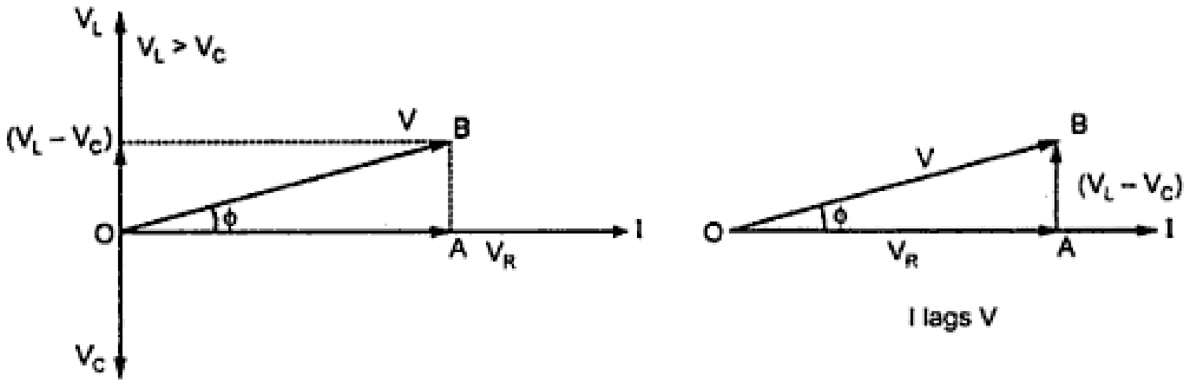


Fig. Phasor diagram and voltage triangle for $X_L > X_C$

From the voltage triangle, $V = \sqrt{(V_R)^2 + (V_L - V_C)^2} = \sqrt{(IR)^2 + (IX_L - IX_C)^2}$

$$= I\sqrt{(R)^2 + (X_L - X_C)^2}$$

$\therefore$ $V = IZ$

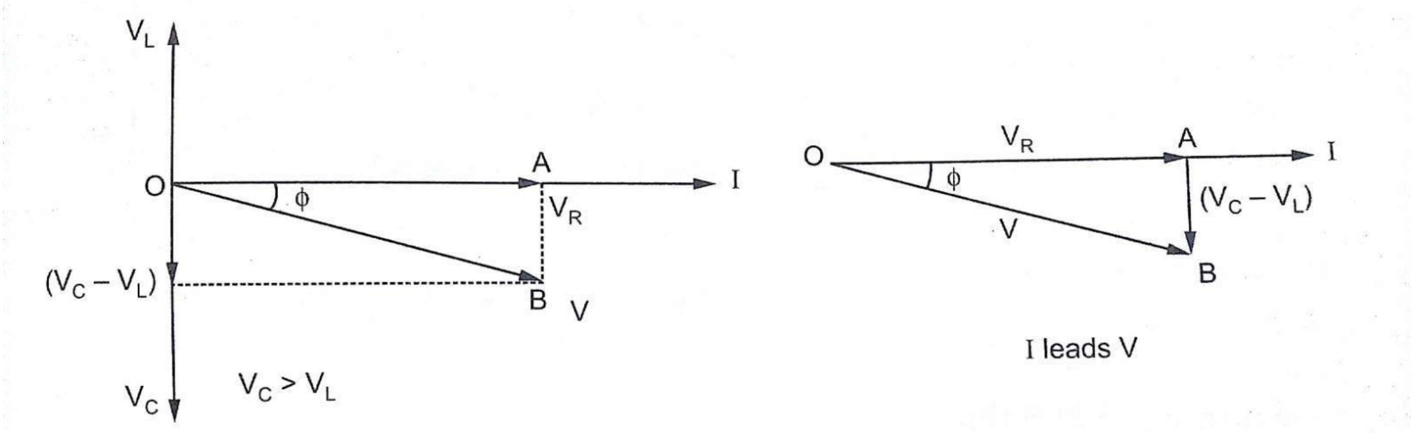
Where $Z = \sqrt{R^2 + (X_L - X_C)^2}$

So, if $v = V_m \sin \omega t$, then $i = I_m \sin(\omega t - \varphi)$ as current lags voltage by angle φ

2. **$X_L < X_C$**

When **$X_L < X_C$,** obviously, IX_L i.e V_L is less than IX_C i.e V_C. So the resultant of V_L and V_C will be directed towards V_C. Current I will lead $(V_C - V_L)$.

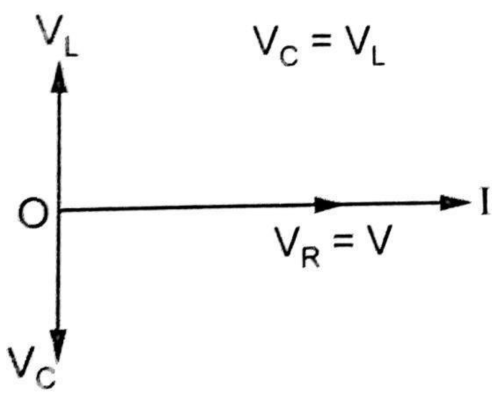
The current is said to be capacitive in nature. The phasor sum of V_R and $(V_C - V_L)$ gives the resultant supply voltage V. This is shown in the fig below.



3. $X_L = X_C$

When $\mathbf{X_L = X_C}$, obviously $\mathbf{V_L = V_C}$. So V_L and V_C will cancel each other and their resultant is zero.

So, $\mathbf{V_R = V}$ in such case and overall circuit is purely resistive in nature. The phasor diagram is shown in fig.



From the phasor diagram V= $\mathbf{V_R = IR= IZ}$ where Z=R

The circuit is purely resistive with unity power factor.

Impedance

In general, for RLC series Circuit impedance is given by,

$$Z = R + jX$$

Where $X = X_L - X_C$ = total reactance of circuit

If $X_L > X_C$, X is positive and circuit is inductive.

If $X_L < X_C$, X is negative and circuit is capacitive.

If $X_L = X_C$, X is zero and circuit is purely resistive.

$$\tan\phi = \left[\frac{X_L - X_C}{R}\right], \quad \cos\phi = \frac{R}{Z} \text{ and } Z = \sqrt{R^2 + (X_L - X_C)^2}$$

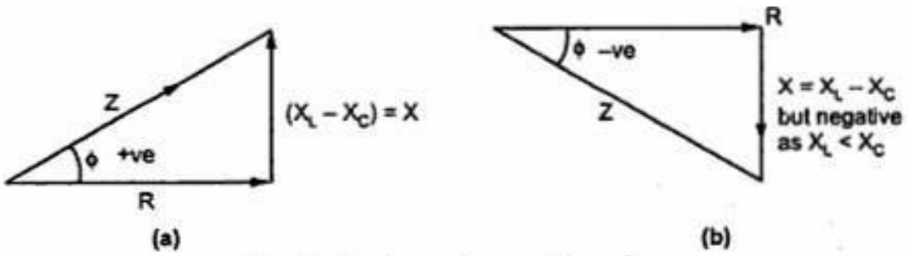
Impedance triangle

The impedance is expressed as,

$$Z = R + jX \quad \text{where} \quad X = X_L - X_C$$

For $X_L > X_C$, φ is positive and impedance triangle is as shown in fig (a)

For $X_L < X_C$, φ is negative and the impedance triangle is as shown in fig (b)



In both cases R= Z cosφ and X= Z sinφ

POWER

The average power consumed by the circuit is,

P_{av} = Average power consumed by R+ Average power consumed by L + Average power consumed by C

But, pure L and C never consume any power.

∴ P_{av} = Power taken by R $= I^2 R = I\,(I\,R) = I\,V_R$

But, $V_R = V\cos\phi$ in both the cases

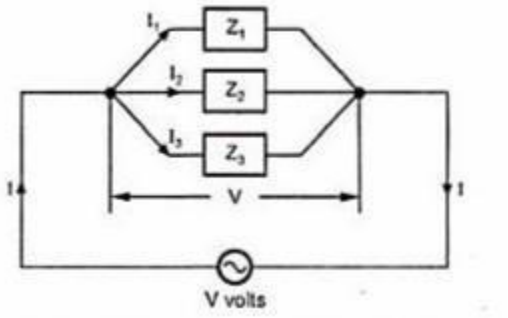
∴ **P = VIcosφ W**

Thus for any condition, in general power can be expressed as **P = VI cosφ**

A.C. Parallel Circuit

A parallel circuit is one in which two or more impedance are connected in parallel across the supply voltage. Each impedance may be a separate series circuit. Each impedance is called branch of the parallel circuit.

The Fig. shows a parallel circuit consisting of three impedances connected in parallel across an ac. supply of V volts.



The current taken by each impedance is different.

Applying Kirchhoff's law, $\bar{I} = \bar{I_1} + \bar{I_2} + \bar{I_3}$ ... (Phasor addition)

$$\therefore \quad \frac{\bar{V}}{\bar{Z}} = \frac{\bar{V}}{\bar{Z_1}} + \frac{\bar{V}}{\bar{Z_2}} + \frac{\bar{V}}{\bar{Z_3}}$$

$$\therefore \quad \frac{1}{\bar{Z}} = \frac{1}{\bar{Z_1}} + \frac{1}{\bar{Z_2}} + \frac{1}{\bar{Z_3}}$$

Where Z is called **equivalent impedance.** This result is applicable for 'n' such impedances connected in parallel.

TWO IMPEDANCE CONNECTED IN PARALLEL

If there are two impedances connected in parallel and if I_T is the total current, then current division rule can be applied to find individual branch currents.

$$\boxed{\begin{aligned} \bar{I}_1 &= \bar{I}_T \times \frac{\bar{Z_2}}{\bar{Z_1} + \bar{Z_2}} \\ \bar{I}_2 &= \bar{I}_T \times \frac{\bar{Z_1}}{\bar{Z_1} + \bar{Z_2}} \end{aligned}}$$

Following are the steps to solve parallel a.c. circuit :

1) The currents in the individual branches are to be calculated by using the relation

$$\bar{I}_1 = \frac{\bar{V}}{\bar{Z}_1}, \quad \bar{I}_2 = \frac{\bar{V}}{\bar{Z}_2}, \ldots, \quad \bar{I}_n = \frac{\bar{V}}{\bar{Z}_n}$$

While the individual phase angles can be calculated by the relation,

$$\tan\phi_1 = \frac{X_1}{R_1}, \tan\phi_2 = \frac{X_2}{R_2}, \ldots, \tan\phi_n = \frac{X_n}{R_n}$$

2) Voltage must be taken as reference phasor as it is common to all branches.
3) Represent all the currents on the phasor diagram and add them graphically or mathematically by expressing them in rectangular form. This is the resultant current drawn from the supply.
4) The phase angle of resultant current I is power factor angle. Cosine of this angle is the power factor of the circuit..

CONCEPT OF ADMITTANCE

Admittance is defined as the reciprocal of the impedance. It is denoted by Y and is **measured in unit siemens or mho.**

Now, current equation for the circuit shown in the Fig. is,

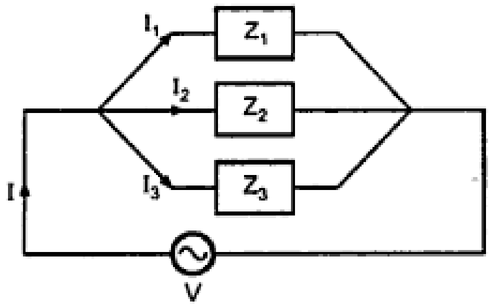
$$\bar{I} = \bar{I}_1 + \bar{I}_2 + \bar{I}_3$$

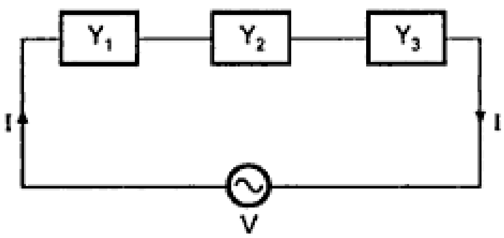
$$\bar{I} = \bar{V} \times \left(\frac{1}{\bar{Z}_1}\right) + \bar{V} \times \left(\frac{1}{\bar{Z}_2}\right) + \bar{V} \times \left(\frac{1}{\bar{Z}_3}\right)$$

$$\overline{VY} = \overline{VY_1} + \overline{VY_2} + \overline{VY_3}$$

$$\therefore \quad \bar{Y} = \bar{Y}_1 + \bar{Y}_2 + \bar{Y}_3$$

where **Y** is the **admittance** of the total circuit. **The three impedances connected in parallel can be replaced by an equivalent circuit, where three admittances are connected in series,** as shown in the Fig. 7.41.





Components of admittance

Consider an impedance given as,

$$Z = R \pm jX$$

Positive sign for inductive and negative for capacitive circuit.

Admittance $\quad Y = \frac{1}{Z} = \frac{1}{R \pm jX}$

Rationalising the above expression,

$$Y = \frac{R \mp jX}{(R \pm jX)(R \mp jX)} = \frac{R \mp jX}{R^2 + X^2}$$

$$= \left(\frac{R}{R^2 + X^2}\right) \mp j\left(\frac{X}{R^2 + X^2}\right) = \frac{R}{Z^2} \mp j\frac{X}{Z^2}$$

$\therefore \quad \boxed{Y = G \mp jB}$

In the above expression, $\quad G = \text{Conductance} = \frac{R}{Z^2}$

and $\quad B = \text{Susceptance} = \frac{X}{Z^2}$

Conductance (G)

It is defined as the ratio of the resistance to the square of the Impedance. It is measured in the unit siemens.

Susceptance (B)

It is defined as the ratio of the reactance to the square of the impedance. It is measured in the unit siemens.

The susceptance is said to be inductive (B_L) if its sign is negative. The susceptance is said to be capacitive (B_C) if its sign is positive.

Note: The sign convention for the reactance and the susceptance are opposite to each other.